4D Trajectory Prediction with Model Predictive Control based on Flight Plan

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Abstract. The prediction of four-dimensional trajectory (4DT), consisting of the three-dimensional position of the aircraft over time, is a key element of Trajectory Based Operation (TBO), the operation concept of next-generation air traffic management, which can identify the predictability of the aircraft, achieve separation for safe and efficient operation, and reduce controller workload. Even in Urban Air Mobility (UAM) environments, the importance of TBO and 4DT predictions is further highlighted to manage complex air flows by predicting the location of aircraft in urban areas with limited space, and to ensure safe distance and efficient operation during aviation. This study proposes a methodology for predicting 4DT, a central element of these TBO. Using the flight plan submitted by the aircraft and the Model Predictive Control (MPC), the optimal control theory, the intention of the aircraft is identified and simulated to generate a 4DT. In this process, speed, altitude limitations in the airspace and aircraft performance were applied as constraints. As a result of the study, the generated 4DT was compared with the actual flight data and the utility was confirmed through the evaluation index. This 4DT generation method is expected to contribute to increasing the efficiency of air traffic management, such as modifying flight plans and strategic separation in the UAM environment and is expected to expand to real-time 4DT generation based on real-time flight data.

Keywords: 4D Trajectory, Trajectory Based Operation, Model Predictive Control, Air Traffic Management

1 Introduction

Trajectory Based Operation (TBO) is a modern approach to air traffic management (ATM) that has emerged by increased air traffic, advances in aircraft technology, and improved computational capabilities of ATM systems. The primary objectives of TBO are to enhance the efficiency and safety of aircraft operations [1]. The critical component of TBO is the four-dimensional trajectory (4DT), which not only includes the temporal and spatial positions of the aircraft, but also its speed, acceleration, and movement. These 4DTs are utilized to predict and adjust aircraft movements with greater precision, playing a significant role in air traffic flow management [2]. The adoption of TBO introduces multiple benefits to air traffic management: it enables efficient management of air traffic while maintaining safe distances between aircraft, optimizes flight time to reduce fuel consumption and the subsequent environmental impact, and reduces uncertainties about flights, thereby improving the efficiency of ATM [3].

In this context, the contribution of our research is to develop an effective methodology for predicting 4DT, using flight plans submitted by aircraft and the Model Predictive Control (MPC), an optimal control theory. Our proposed method, by incorporating the constraints of speed and altitude limitations in airspace and aircraft performance, seeks to enhance the accuracy of 4DT predictions. This approach to 4DT prediction has significant implications for air traffic management, particularly within Urban Air Mobility (UAM) environments, facilitating the modification of flight plans, ensuring strategic separation, and paving the way for real-time 4DT generation based on real-time flight data.

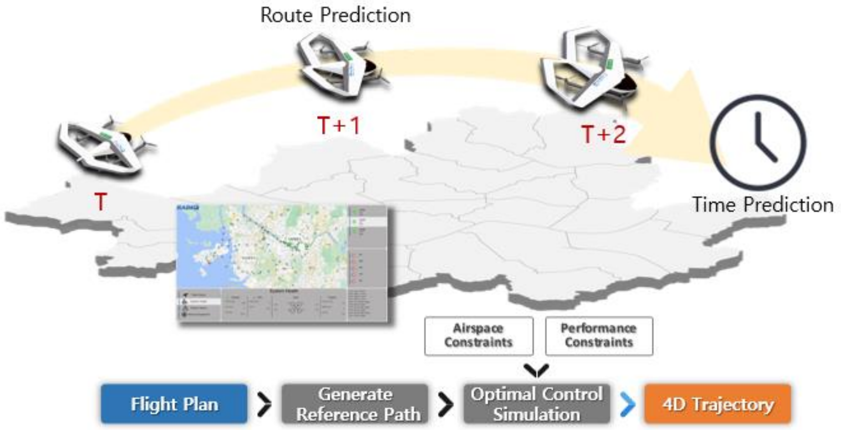

Fig. 1. Concept of 4DT Prediction

2 Literature Review and Background

2.1 UAM and the Importance of 4D Trajectory Generation

UAM has been attracting the attention of stakeholders due to its potential to play a crucial role in future urban transportation systems.[4]–[6] A vital aspect of UAM operations is the generation of safe and efficient 4DT, which include latitude, longitude, altitude, and time. These trajectories are imperative for navigating through complex and dynamic urban airspace, while ensuring adherence to airspace regulations, minimizing conflicts, and maintaining efficient operations.

In UAM operations, the UAM operator is a key player, managing specific UAM activities. UAM service providers, offering support services, also play an essential role. Route planning and deconfliction of flight paths are critical responsibilities for both operators and service providers to ensure the safety and efficiency of UAM flights. The concept of Operational Volumes (OVs) [4], [7], [8] introduces 4D regions of airspace, where time is an additional dimension. These OVs define designated regions where UAM vehicles can operate, creating a contractual agreement between aircraft operators and airspace management. Compliance with OVs ensures safe and reliable task execution for UAM operations.

Despite the fundamental importance of 4D trajectory generation in UAM, literature in this specific domain is relatively scarce. Researchers often derive insights from cooperative 3D trajectory generation and simultaneous arrival methods. Trajectory descriptions can be either continuous or discrete, with continuous trajectories using mathematical expressions such as Dubbin’s paths [9], Pythagorean Hodograph (PH) [10] curves, and tensor field guidance [11]. Discrete trajectories, such as the A*, D*, and iADA* search algorithms [12]–[14], are preferred for handling the NP-hard nature of trajectory generation.

2.2 Methods for 4D Trajectory Prediction and Application

TBO represents a transformation in air traffic management towards a more efficient future, with a core focus on 4DT, encompassing latitude, longitude, altitude, and time. This shift towards individual aircraft trajectories over predefined routes boosts operational efficiency and has highlighted the importance of 4D trajectory prediction [1] – [3].

4D trajectory prediction methods can be categorized into three types: state prediction models, kinematic models, and machine learning models [15]. State prediction models, using physics and mathematical relationships, can accurately forecast short-term aircraft states, but struggle with complex aircraft maneuvers. In contrast, kinematic models, which apply Newton's laws of motion, predict trajectories based on initial state and environmental conditions, making them suitable for scenario-based simulations. Machine learning models also contribute to 4D trajectory prediction by training algorithms using historical flight data. This model has shown to be effective where substantial past flight data is available and find their application mainly in commercial flight trajectory prediction [16].

In our study, we propose a fusion of state prediction and kinematic models in a state estimation model that predicts future states based on the current state of the aircraft. We effectively integrate the kinematic model and the state estimation model by utilizing the point mass model of the aircraft and the MPC theory. This novel approach addresses the limitations of traditional state prediction models by allowing us to capture aircraft maneuvers and predict long-term flight trajectories.

Our primary aim is to devise a safe and efficient 4D trajectory generation method for UAM operations within urban areas. We consider uncertainties in UAM performance data and planned flight paths using the MPC algorithm. Our approach seeks to address the unique challenges posed by dynamic and complex urban airspace and contribute to the integration of UAM into urban transportation systems. Our method's efficacy has been assessed through case studies using existing domestic flight data, as UAM flight data were not available at the time of our research.

3 Concept of 4D Trajectory Prediction

3.1 Route Prediction

According to ICAO's definition, an aircraft's flight plan contains information about the aircraft's identifier, flight rules, departure, arrival, planned routes, cruise speed, altitude, and fuel. And it is submitted to air traffic control before takeoff.[17]. The flight plan is a document that identifies the route of the aircraft, and the air traffic management system tracks the position of the aircraft through the flight plan and provides services as needed. Since the route consists of a set of waypoints expressed in latitude and longitude, the flight plan represents a series of points that the aircraft will pass. This flight plan allows the ATC to predict the path of the aircraft first.

Many of the UAM operational concepts studied also envision UAM to carry out the creation and submission of flight plans in a form similar to the current air control system.[18]-[20] The aircraft's fleet operator will submit a flight plan containing the same information as the current flight plan to the Provider of Service for UAM (PSU) and monitor the UAM based on this flight plan. Due to UAM flying at low altitudes in urban areas, the UAM's flight route is expected to be in the form of a corridor that provides a specific volume that can be flown. It is easier to predict routes through flight plans compared to conventional aviation. This is because UAM has a shorter flight time and corridor-type routes, reducing the likelihood of emergency situations like rapid weather changes.

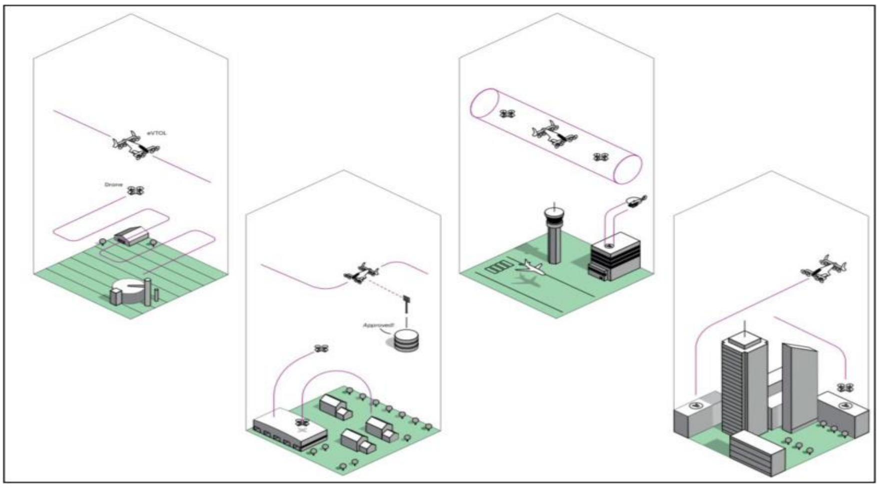

Fig. 2. Airspace structures by Airbus (left to right): Basic Flight, Free Route, Corridors, and Fixed Route

3.2 Time Prediction

The flight path, the corridor, was not straight. Therefore, the aircraft performs control to avoid deviating from the predetermined corridor while aiming for the next target point (waypoint). The control of the aircraft is carried out according to the aircraft's performance and operational restrictions, as well as the constraints of the corridor. Due to this control, the speed, position, and attitude of the aircraft change, making it difficult to predict the position over time. On the other hand, if one predicts how the aircraft will follow and control a set route, the position of the aircraft over time can be predicted more accurately.[21]

The path-tracking problem of aircraft is a complex optimal control problem that must be solved efficiently by considering various factors.[22], [23] For example, an aircraft's pilot or autopilot system performs optimal control by considering errors with the planned route, the aircraft's performance, and the value of the jerk. One of the most effective ways to solve these problems is model predictive control. MPC is a method of predicting future states based on the current state and deriving and performing optimal control values based on it.[24],[25] This approach does not merely follow an immediate trajectory; instead, it predicts a smooth and efficient future trajectory, mirroring real-world flight paths. In addition, MPC can explicitly include the aircraft's performance and airspace constraints as constraints of control, making it easy to generate real-world flight-able trajectories.

Therefore, Using MPC theory and flight plan information, it is possible to predict both the path and time, enabling the generation of a 4DT model for the aircraft. This paper assumes a general situation where there is no problem with the planned path and predicts a 4DT by considering that the aircraft follows the planned path and performs optimal control with restrictions on the performance and airspace of the aircraft.

4 Methodology for 4D trajectory Prediction with MPC

4.1 Aircraft Point Mass Model

To derive control values to predict 4DT using the MPC theory, we used the 6-DOF point mass model of an aircraft. The creation of a 4DT requires a kinematic perspective that shows where it is at any given point in time, rather than the detailed posture and dynamics of the aircraft. Many 4DT research and fast simulations utilize such a point mass model.[26],[27] The motion model of the modeled aircraft is as per the following equations (1) to (9).

$$\dot{\mathrm{x}} = \mathrm{V} \cos \gamma \cos \psi \tag{1}$$

$$\dot{\mathrm{y}} = \mathrm{V} \cos \gamma \sin \psi \tag{2}$$

$$\dot{\mathrm{h}} = -\mathrm{V} \sin \gamma \tag{3}$$

$$\dot{\gamma} = \frac{g}{V} \cos \gamma - \frac{g}{V} \cos \phi \, n_L \tag{4}$$

$$\dot{\psi} = -\frac{g}{V}\frac{\sin\phi}{\cos\gamma}n_L \tag{5}$$

$$\dot{\phi} = \omega \tag{6}$$

$$\dot{V} = g\sin\gamma + gn_T \tag{7}$$

$$n_L = \frac{L}{mg} \tag{8}$$

$$n_T = \frac{T-D}{mg} \tag{9}$$

The aircraft kinematic model is based on the ENU (East-North-Up) coordinates. The state values of the aircraft are x, y, h, γ, ψ, ϕ and V, where x, y and h are each the ENU coordinate values. γ represents the aircraft's flight path angle, while ψ represents the heading angle of the aircraft based on ENU coordinates. ϕ is the bank angle of the aircraft. V represents the speed of the aircraft. The control values of the aircraft were set as n_L, n_T and ω. n_L and n_T correspond to the lift and thrust forces relative to the aircraft's weight, respectively, and ω corresponds to the angular velocity for the bank angle. With this approach, it is possible to conveniently perform simulations without being confined to modeling a specific aircraft, by limiting control inputs through constraints. To transform the equations for the MPC into discrete time, the Euler method was employed, which modifies the system based on time step.

4.2 Reference State for Control

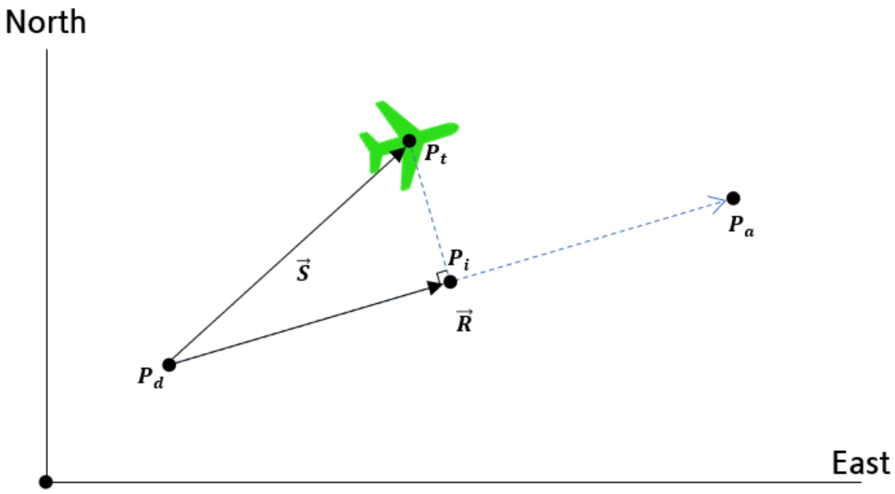

Fig. 3. Reference Point of Aircraft flying in a corridor

Optimizing control with MPC derives control values to maintain the system in an ideal state. The ideal state for an aircraft can be considered as the state when it flies at the planned speed without deviation from the path indicated in the flight plan. Therefore, the ideal state values (reference values) that become the goal of control can be derived through the flight plan.

Denote the 2-dimensional position of the aircraft at some time t as P_t. The ideal position where the aircraft should have been located according to the flight plan can be viewed as the point P_i on the straight line formed by the previous waypoint P_d and the target waypoint P_a. The value of point P_i can be obtained by projecting the vector S, formed by points P_d and P_t, onto the vector R, formed by points P_d and P_a. Similarly, the heading direction that the aircraft should aim for is also the angle of vector R.

The reference values for speed and altitude were set as the planned values in the current section of the flight plan for control. For the cruising section, the planned cruising speed and altitude were set as reference values, and for the descent and ascent sections, the altitude and speed of the route according to SID & STAR were set as reference values. The bank angle and flight path angle were not included in the calculation of the target or cost function for control, and their values were only limited in the form of constraints in the optimization to be discussed later.

4.3 Cost Function and Constraint for MPC

$$\min \mathrm{J} = \sum_{k=1}^{N}\left(x(k) - x_{\mathrm{REF}}(k)\right)^{\mathrm{T}} Q\left(x(k) - x_{\mathrm{REF}}(k)\right) \tag{10}$$

$$+ \sum_{k=1}^{N} \Delta u(k)^{\mathrm{T}} \mathrm{R} \Delta u(k) \tag{11}$$

$$-S \sum_{k=1}^{N} V(k)^2 \tag{12}$$

$$subject\ to \qquad x_{i+1} = f(x_i, u_i) \quad \text{for} \quad i = 0, \dots, N-1 \tag{13}$$

$$x_i \geq x_{\min} \quad i = 1, \dots, N \tag{14}$$

$$x_i \leq x_{\max} \quad i = 1, \dots, N \tag{15}$$

$$\mathrm{u}_i \geq \mathrm{u}_{\min} \quad i = 1, \dots, N \tag{16}$$

$$\mathrm{u}_i \leq \mathrm{u}_{\max} \quad i = 1, \dots, N \tag{17}$$

In conventional applications of MPC, the objective function for optimization is often represented as a sum of quadratic forms, with the goal of minimizing both the deviation between the current state and reference state(10), as well as the changes in control inputs.(11) However, in this paper, we expanded upon the traditional objective function to consider the intent of reaching the destination as quickly as possible. To achieve this, the sum of the squares of the aircraft's speed at each step was incorporated into the objective function.(12) In this context, x represents the state vector of the aircraft, x_{REF} denotes the reference state, Δu stands for the change in control input u, and V is the aircraft's velocity.

We have imposed several constraints on the state.(13)-(17) First, the constraints ensure that the designated motion equations are satisfied. Furthermore, there are constraints set for the flight path and bank angle. Considering the performance characteristics of the aircraft, we also implemented a speed constraint and a tolerance for the planned flight speed based on the flight plan. In addition, considering the aircraft's performance, constraints have been established for the control inputs.

5 Result

On Monday, August 7th, 2023 (GMT+9), we compared the flight trajectory data and prediction results for the Gimpo-Jeju route, BX8041 flight. Among the entire route, we selected flight trajectory data from the approach phase, where there are noticeable changes in altitude and speed, and distinct turns, as these factors can cause errors in 4-dimensional trajectory predictions. We assumed a flight plan based on the STAR procedure of Jeju Airport, and the impact of wind was not considered in the prediction. The data from the moment the aircraft passed the DOTOL point was used as the initial condition for the prediction, and the point at which the aircraft arrived at the PC624 point was set as the end of the prediction. The actual flight time for this segment was 672 seconds, but the predicted flight time was calculated as 693 seconds.

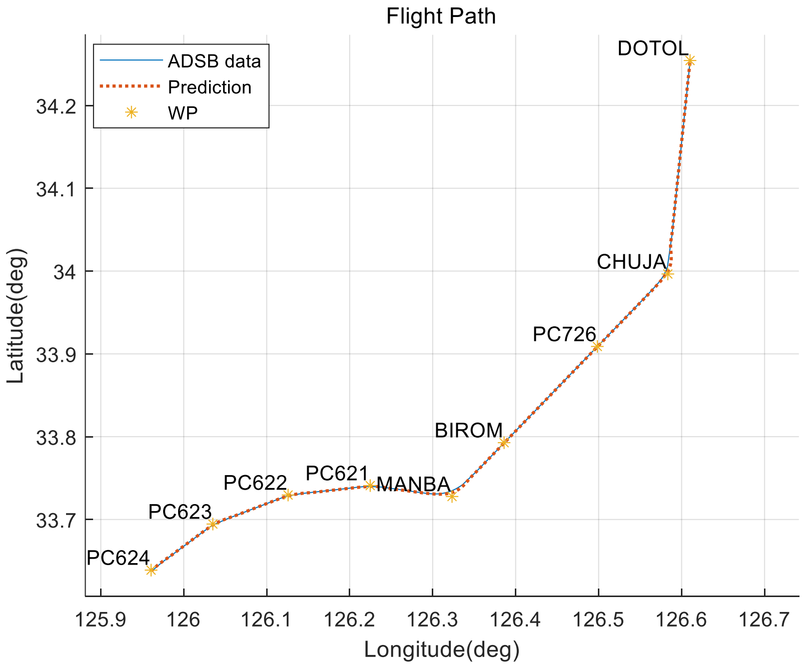

Fig. 4. 2D(latitude, longitude) Trajectory Data of BX8041 and Prediction Result

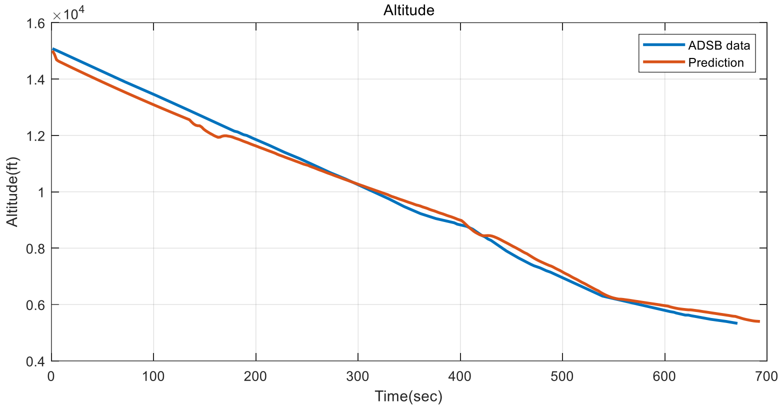

Fig. 5. Altitude Data of BX8041 and Prediction Result

Fig 6 and Table 1 present the calculation results concerning the accuracy of the predicted trajectory. The graph delineates the error between the Cross Track Error (CTE) and the arrival time to each waypoint. CTE showed notable discrepancies at turning points, particularly at CHUJA and MANBA. The most pronounced differences in CTE were observed at T+158 and T+397, which are moments right before arriving at CHUJA and MANBA, respectively. At these instances, the CTE values recorded were 310m and 242m. This significant deviation in the CTE appears to be due to the MPC-based prediction calculating a trajectory that resulted in a Fly-by closer to the waypoint than the actual flight path. Apart from these specific points, the CTE for other segments was observed to be around 50m. Regarding the error in arrival time, T+0 denotes the time of departure from DOTOL, with the error progressively accumulating and then decreasing again as the route continues.

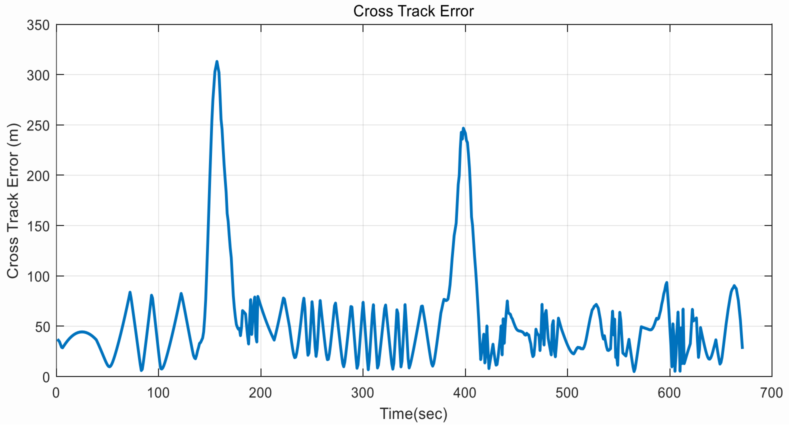

Fig. 6. BX8041 Trajectory Contrast Cross-Track Error

Table 1. Arrival time Error of Each Waypoint

	ADSB data	Prediction	Error
CHUJA	T +167s	T +171s	4s
PC726	T +243s	T +250s	7s
BIROM	T +344s	T +361s	17s
MANBA	T +404s	T +423s	19s
PC621	T +467s	T +489s	22s
PC622	T +535s	T +557s	22s
PC623	T +604s	T +625s	21s
PC624	T +672s	T +693s	19s

6 Conclusion

In this paper, we have proposed a method for predicting an aircraft's 4-dimensional trajectory using flight planning and optimal control theory. Especially, this method can be effectively applied to UAM characterized by short flight times and circuitous routes. The importance of such 4-dimensional trajectory predictions is expected to be emphasized further in high-density flight environments.

We compared the actual flight trajectory data with our predictions, analyzing discrepancies in time and position between the actual and predicted data. Accuracy for positions was verified using cross-track error, and the accuracy of time prediction was determined by comparing the difference in arrival times at each waypoint.

Importantly, we anticipate that by fine-tuning the MPC parameters and incorporating more sophisticated modeling that includes external factors like wind, the accuracy of trajectory predictions can be further enhanced. Additionally, it is expected that the methodology presented in this research can be expanded to be applicable in vertical takeoff or multi-copter flight modes of UAM.

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References

1. FAA/EUROCONTROL: Common TP Structure and Terminology in Support of SESAR & NextGen; Technical Report FAA/EUROCONTROL: Brussels, Belgium, 2010.
2. ICAO: Global Air Traffic Management Operational Concept; ICAO: Montreal, QC, Canada, 2005.
3. Ruiz, Sergio, Javier Lopez Leones, and Andrea Ranieri. "A novel performance framework and methodology to analyze the impact of 4D trajectory based operations in the future air traffic management system." Journal of Advanced Transportation 2018 (2018): 1-17.

4. B. Hill and D. DeCarme, "Urban Air Mobility (UAM) Vision Concept of Operations (ConOps) UAM Maturity Level (UML)-4 Overview," Accessed: Jul. 20, 2022. [Online]. Available: https://ntrs.nasa.gov/citations/20210010443
5. D. P. Thipphavong et al., "Urban Air Mobility Airspace Integration Concepts and Considerations," in 2018 Aviation Technology, Integration, and Operations Conference, in AIAA AVIATION Forum. American Institute of Aeronautics and Astronautics, 2018. doi: 10.2514/6.2018-3676.
6. K. O. Ploetner et al., "Long-term application potential of urban air mobility complementing public transport: an upper Bavaria example," CEAS Aeronaut. J., vol. 11, no. 4, pp. 991–1007, Dec. 2020, doi: 10.1007/s13272-020-00468-5.
7. "Concept of operations for European UTM systems – Extension for urban air mobility (CORUS-XUAM)." https://www.eurocontrol.int/project/corus-xuam (accessed Aug. 01, 2023).
8. "Aerospace | Free Full-Text | UTM Architecture and Flight Demonstration in Korea." https://www.mdpi.com/2226-4310/9/11/650 (accessed Aug. 01, 2023).
9. M. Shanmugavel, A. Tsourdos, R. Zbikowski, and B. White, "Path Planning of Multiple UAVs Using Dubins Sets." 2005. doi: 10.2514/6.2005-5827.
10. M. Shanmugavel, A. Tsourdos, R. Zbikowski, and B. White, "3D Path Planning for Multiple UAVs Using Pythagorean Hodograph Curves," in AIAA Guidance, Navigation and Control Conference and Exhibit, in Guidance, Navigation, and Control and Co-located Conferences. American Institute of Aeronautics and Astronautics, 2007. doi: 10.2514/6.2007-6455.
11. S. C. Degen, L. M. Alvarez, J. J. Ford, and R. A. Walker, "Tensor Field guidance for time-based waypoint arrival of UAVs by 4D trajectory generation," in 2009 IEEE Aerospace conference, Mar. 2009, pp. 1–7. doi: 10.1109/AERO.2009.4839605.
12. O. Nahum, "Multi-Objective Vehicle Routing Problems with Time Windows: A Vector Evaluated Artificial Bee Colony Approach," Int. J. Comput. Inf. Technol., vol. 3, pp. 41–47, Jan. 2014.
13. A. Stentz, "Optimal and efficient path planning for partially-known environments," in Proceedings of the 1994 IEEE International Conference on Robotics and Automation, May 1994, pp. 3310–3317 vol.4. doi: 10.1109/ROBOT.1994.351061.
14. A. A. Maw, M. Tyan, and J.-W. Lee, "iADA*: Improved Anytime Path Planning and Replanning Algorithm for Autonomous Vehicle," J. Intell. Robot. Syst., Aug. 2020, doi: 10.1007/s10846-020-01240-x.
15. W. Zeng, X. Chu, Z. Xu, Y. Liu, and Z. Quan, "Aircraft 4D Trajectory Prediction in Civil Aviation: A Review," Aerospace, vol. 9, no. 2, Art. no. 2, Feb. 2022, doi: 10.3390/aerospace9020091.
16. N. Schimpf, E. J. Knoblock, Z. Wang, R. D. Apaza, and H. Li, "A Generalized Approach to Aircraft Trajectory Prediction via Supervised Deep Learning."
17. ICAO, Annex II to the convention on international civil aviation: Aerodromes. In Rules of the Air ICAO, 10th ed.; ICAO: Quebec, QC, Canada, July 2005.
18. FAA, Concept of Operations v1.0. Urban Air Mobility (UAM), FAA, (2020).
19. Youngjae Lee, Tae Ho Kwag, Gu Moon Jeong, Jae Hyun Ahn, Bong Cheul Chung, Jae-Woo Lee.: Flight Routes Establishment Through the Operational Concept Analysis of Urban Air Mobility System. Journal of the Korean Society for Aeronautical & Space Sciences, 48(12), (2020).
20. Bauranov, Aleksandar, and Jasenka Rakas.: Designing airspace for urban air mobility: A review of concepts and approaches. Progress in Aerospace Sciences, 125, (2021).
21. Garcia-Chico, J; Vivona, R; Cate, K.: Characterizing intent maneuvers from operational data: Step towards trajectory prediction uncertainty estimation. In Proceedings of the AIAA

Guidance, Navigation and Control Conference and Exhibit, Honolulu, HI, USA, 18–21 (2008).

22. Hyeon Jun Lee, Dasom Kim, SuJin Kang, BoHoon Moon, Nan Lao Ywet, Aye Aye Maw, Jae-Woo Lee.: UAM's Elementary/Medium Phase of Autonomous Flight System Using Frenet Optimal Trajectory Planning. Proceeding of The Korean Society for Aeronautical and Space Sciences Spring Conference, 142-143. (2022).
23. Sang Pyuk Lee, Min Gyu Lim, Jun Young Ahn, Chang Joo Kim.: Research on Optimal Control of Fixed-Wing Aircraft Tactical Maneuver Using Direct Dynamic Simulation Approach. Proceeding of The Korean Society for Aeronautical and Space Sciences Spring Conference, 191-192. (2022).
24. T. Baca, D. Hert, G. Loianno, M. Saska and V. Kumar, "Model Predictive Trajectory Tracking and Collision Avoidance for Reliable Outdoor Deployment of Unmanned Aerial Vehicles," 2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Madrid, Spain, 2018, pp. 6753-6760.
25. Taewon Uhm, Taesan Park, Seungkeun Kim. (2020). Autonomous Path Generation Method of Obstacle Avoidance for Small UAV using Linear Model Predictive Control. Proceeding of The Korean Society for Aeronautical and Space Sciences Spring Conference, 327-328.
26. Zhang, J.; Liu, J.; Hu, R.; Zhu, H. Online four-dimensional trajectory prediction method based on aircraft intent updating. Aerospace Science and Technology, 77, 774–787 (2018).
27. Weitz, Lesley A. "Derivation of a point-mass aircraft model used for fast-time simulation." MITRE Corporation (2015).